Unit 5: Polygons, Angles and Parallel Lines

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
- Understand 'regular' and 'irregular' as applied to polygons;
- Understand the proof that the angle sum of a triangle is $180^{\circ}$, and derive and use the sum of angles in a triangle;
- Use symmetry property of an isosceles triangle to show that base angles are equal;
- Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is $360^{\circ}$; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is $360^{\circ}$;


## Mathematics Department

## Key stage 4: Higher

- Calculate the angles of regular polygons and use these to solve problems;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Use angle facts to demonstrate how shapes would 'fit together', and work out interior angles of shapes in a pattern.
- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance-time and velocity-time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.


## Mathematics Department

## Key stage 4: Higher

Unit 6: Co-ordinate Geometry

- Plot and draw graphs of $y=a, x=a, y=x$ and $y=-x$, drawing and recognising lines parallel to axes, plus $y=x$ and $y=-x$;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form $y=m x+$ $c$ correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and $y$ intercept of a linear graph given by equations of the form $y=m x+c$;
- Find the equation of a straight line from a graph in the form $y=m x+c$;
- Plot and draw graphs of straight lines of the form $y=m x+c$ with and without a table of values;
- Sketch a graph of a linear function, using the gradient and $y$-intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation $a x+b y=c$;
- Find the equation of a straight line from a graph in the form $a x+b y=c$;
- Plot and draw graphs of straight lines in the form $a x+b y=c$;
- Interpret and analyse information presented in a range of linear graphs:
- use gradients to interpret how one variable changes in relation to another;
- find approximate solutions to a linear equation from a graph;
- identify direct proportion from a graph;
- find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when $y=m x+c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of $m$ and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$.
- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Interpret graphs of quadratic functions from real-life problems;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function $y=\frac{1}{x}$ with $x \neq 0$ using tables of values;
- Draw circles, centre the origin, equation $x^{2}+$
- $y^{2}=r^{2}$.


## End of year assessment <br> This will include all topics covered

Misconceptions will be addressed as part of the end of year assessment review process

- calculate exactly with fractions, surds and multiples of $\pi$; simplify surd expressions involving squares $(e . g . \sqrt{ } 12=\sqrt{ }(4 \times 3)=\sqrt{ } 4$ $\times \sqrt{ } 3=2 \sqrt{ } 3$ ) and rationalise denominators.
- use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate.
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology.
- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding.
- apply and interpret limits of accuracy, including upper and lower bounds.
- change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts.
- use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description.
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment.
- identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres.
- use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.).
- know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of


Unit 8: Constructions and Transformations

Autumn
2
cuboids and other right prisms (including cylinders).

- know the formulae: circumference of a circle $=$ $2 \pi r=\pi d$, area of a circle $=\pi r^{2}$; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids.
- calculate arc lengths, angles and areas of sectors of circles.
- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations - know that that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid - know to include the mirror line as a simple algebraic equation, $x=a, y=$ $a, y=x, y=-x$ and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also $y=x$ and $y=-x$;
- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;


## Mathematics Department

## Key stage 4: Higher

- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;
- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.
- Draw 3D shapes using isometric grids;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
- bisect a given angle;
- construct a perpendicular to a given line from/at a given point;
- construct angles of $90^{\circ}, 45^{\circ}$;
- perpendicular bisector of a line segment;
- Construct:
- a region bounded by a circle and an intersecting line;
- a given distance from a point and a given distance from a line;
- equal distances from two points or two line segments;
- regions which may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line
- Factorise quadratic expressions in the form $a x^{2}$ $+b x+c$;
- Set up and solve quadratic equations;
- Solve quadratic equations by factorisation and completing the square; SCHOOL


## Mathematics Department

## Key stage 4: Higher



- Solve quadratic equations that need rearranging;
- Solve quadratic equations by using the quadratic formula;
- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
- linear / linear, including where both need multiplying;
- linear / quadratic;
- linear $/ x^{2}+y^{2}=r^{2}$;
- Set up and solve a pair of simultaneous equations in two variables for each of the above scenarios, including to represent a situation;
- Interpret the solution in the context of the problem;


## Mathematics Department

## Key stage 4: Higher



- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- List all outcomes for single events, and combined events, systematically;
- Draw sample space diagrams and use them for adding simple probabilities;
- Know that the sum of the probabilities of all outcomes is 1 ;
- Use $1-p$ as the probability of an event not occurring where $p$ is the probability of the event occurring;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Find a missing probability from a list or two-way table, including algebraic terms;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events;
- Use a two-way table to calculate conditional probability;
- Use a tree diagram to calculate conditional probability;
- Use a Venn diagram to calculate conditional probability;


## Mathematics Department

## Key stage 4: Higher

|  | - Compare experimental data and theoretical probabilities; <br> - Compare relative frequencies from samples of different sizes. |  |
| :---: | :---: | :---: |
| Unit 11: Multiplicative Reasoning | - Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when $A: B$ are in the ratio $3: 5, A$ is $\frac{3}{5} B$. When $4 a=7 b$, then $a=\frac{7 b}{4}$ or $a: b$ is $7: 4$; <br> - Solve proportion problems using the unitary method; <br> - Work out which product offers best value and consider rates of pay; <br> - Work out the multiplier for repeated proportional change as a single decimal number; <br> - Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation; <br> - Understand and use compound measures and: <br> - convert between metric speed measures; <br> - convert between density measures; <br> - convert between pressure measures; <br> - Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question); <br> - Calculate an unknown quantity from quantities that vary in direct or inverse proportion; <br> - Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of $k$ in $y=k x$; <br> - Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19); | End of Unit test |

## Mathematics Department

## Key stage 4: Higher



| Term | Topic | Learning Outcomes | Assessment |
| :--- | :--- | :--- | :--- |
|  |  |  | Understand and use SSS, SAS, ASA and RHS <br> conditions to prove the congruence of triangles using <br> formal arguments, and to verify standard ruler and <br> pair of compasses constructions; |

## Mathematics Department

## Key stage 4: Higher

- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.
- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) $y=\sin x, y=\cos x$ and $y=\tan x$ for angles of any size.
- Know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta=0^{\circ}$, $30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$ and exact value of $\tan \theta$ for $\theta$ $=0^{\circ}, 30^{\circ}, 45^{\circ}$ and $60^{\circ}$ and find them from graphs.
- Apply to the graph of $y=\mathrm{f}(x)$ the transformations $y=$ $-\mathrm{f}(x), y=\mathrm{f}(-x)$ for sine, cosine and tan functions $\mathrm{f}(x)$.
- Apply to the graph of $y=\mathrm{f}(x)$ the transformations $y=$ $\mathrm{f}(x)+a, y=\mathrm{f}(x$

End of Unit Test for sine, cosine and tan functions $f(x)$.

Summer 2 Unit 13: Advanced Trigonometry

- Know and apply Area $=\frac{1}{2} a b \sin C$ to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.


# End of year assessment <br> This will include all topics covered <br> Misconceptions will be addressed as part of the end of year assessment review process 

Autumn 1
Unit 14: Statistics

- Specify the problem and plan:
- decide what data to collect and what analysis is needed;
- understand primary and secondary data sources;
- consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it;
- Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative (see note);
- Use statistics found in all graphs/charts in this unit to describe a population;
- Know the appropriate uses of cumulative frequency diagrams;
- Construct and interpret cumulative frequency tables, cumulative frequency graphs/diagrams and from the End of Unit Test graph:
- estimate frequency greater/less than a given value;
- find the median and quartile values and interquartile range;


## Mathematics Department

## Key stage 4: Higher

Unit 15: Advanced Algebra

- Compare the mean and range of two distributions, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
- Produce box plots from raw data and when given quartiles, median and identify any outliers;
- Know the appropriate uses of histograms;
- Construct and interpret histograms from class intervals with unequal width;
- Use and understand frequency density;
- From histograms:
- complete a grouped frequency table;
- understand and define frequency density;
- Estimate the mean and median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval
- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots and $y$ intercept, turning point;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Expand the product of more than two linear expressions;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Solve simultaneous equations graphically:
- find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
- find graphically the intersection points of a given straight line with a circle;


## Mathematics Department



- solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
- for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^{2}-3 x-10<0$ as $\{x$ : $3<x<5\}$;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

| Term | Topic | Learning Outcomes | Assessment |
| :---: | :---: | :---: | :---: |

Unit 16: Circle Theorems

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
- Prove and use the facts that:
- the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
- the angle in a semicircle is a right angle;
- the perpendicular from the centre of a circle to a chord bisects the chord;
- angles in the same segment are equal;
- alternate segment theorem;
- opposite angles of a cyclic quadrilateral sum to $180^{\circ}$;


## Mathematics Department

- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
- circle theorems;
- isosceles triangles (radius properties) in circles;
- the fact that the angle between a tangent and radius is $90^{\circ}$;
- 
- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
- Find the equation of a tangent to a circle at a given point, by:
- finding the gradient of the radius that meets the circle at that point (circles all centre the origin);

Mock Exam

- finding the gradient of the tangent perpendicular to it;
- using the given point;
- Recognise and construct the graph of a circle using $x^{2}$ $+y^{2}=r^{2}$ for radius $r$ centred at the origin of coordinates.
- Rationalise the denominator involving surds;
- Simplify algebraic fractions;
- Multiply and divide algebraic fractions;
- Solve quadratic equations arising from algebraic fraction equations;
- Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;
- Change the subject of a formula such as $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, where all variables are in the denominators;
- Solve 'Show that' and proof questions using consecutive integers $(n, n+1)$, squares $a^{2}, b^{2}$, even numbers $2 n$, odd numbers $2 n+1$;

Unit 18: Vectors and Geometric Proof

- Use function notation;
- Find $\mathrm{f}(x)+\mathrm{g}(x)$ and $\mathrm{f}(x)-\mathrm{g}(x), 2 \mathrm{f}(x), \mathrm{f}(3 x)$ etc algebraically;
- Find the inverse of a linear function;
- Know that $\mathrm{f}^{-1}(x)$ refers to the inverse function;
- For two functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$, find $\mathrm{gf}(x)$.
- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
- Understand that $2 \mathbf{a}$ is parallel to $\mathbf{a}$ and twice its length, and that $\mathbf{a}$ is parallel to -a in the opposite direction.
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
- Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
- Find the length of a vector using Pythagoras' Theorem.
- Calculate the resultant of two vectors.
- Solve geometric problems in 2D where vectors are divided in a given ratio.
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel

|  | Term | Topic | Learning Outcomes | Assessment |
| :---: | :---: | :---: | :---: | :---: |
|  | Spring 2 | Unit 19: Proportion | - Recognise, sketch and interpret graphs of the reciprocal function $y=\frac{1}{x}$ with $x \neq 0$ <br> - State the value of $x$ for which the equation is not defined; <br> - Recognise, sketch and interpret graphs of exponential functions $y=k^{x}$ for positive values of $k$ and integer values of $x$; <br> - Use calculators to explore exponential growth and decay; |  |

## Mathematics Department

- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of $\mathrm{f}(x)+a$, or $\mathrm{f}(x-a)$ :
- apply to the graph of $y=\mathrm{f}(x)$ the transformations $y=$ $-\mathrm{f}(x), y=\mathrm{f}(-x)$ for linear, quadratic, cubic functions;
- apply to the graph of $y=f(x)$ the transformations $y=$
$\mathrm{f}(x)+a, y=\mathrm{f}(x+a)$ for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance-time and velocity-time graphs:
- for a non-linear distance-time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
- for a non-linear velocity-time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.
- Recognise and interpret graphs showing direct and indirect proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for $x$ squared and $x$ cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use $y=k x$ to solve direct proportion problems, including questions where students find $k$, and then use $k$ to find another value;
- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.


## Exam dates:

## Paper 1: 16 ${ }^{\text {th }}$ May 2024

Paper 2: $3^{\text {rd }}$ June 2024
Paper 3: 10th June 2024

